Graph theory: Introduction

Contents

Contents

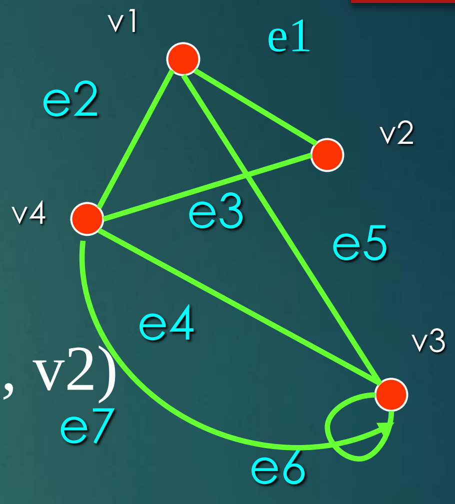
Introduction to Graphs

Definition: A graph is collection of points called **vertices** & collection of lines called **edges** each of which joins either a pair of points or single points to itself.

Mathematically graph G is an ordered pair of (V, E)

Each edge e_{ij} is associated with an ordered pair of vertices (V_i, V_j).

Introduction to Graphs



Graph G

In Fig. G has graph 4 vertices namely

v1, v2, v3, v4& 7 edges

Namely e1, e2, e3, e4, e5, e6, e7 Then e1=(v1, v2)

Similarly for other edges.

In short, we can represent G=(V,E) where V=(v1, v2, v3, v4) & E=(e1, e2, e3, e4, e5, e6,e7)

Self Loops & Parallel Edges

Definition: If the end vertices V_i & V_j of any edge e_{ij} are same, then edge eij called as **Self Loop.**

For Example, In graph G, the edge $e_6 = (v_3, v_3)$ is self loop.

Definition: If there are more than one edge is associated with given pair of vertices then those edge called as **Parallel or Multiple edge.**

For Example, In graph G, e_4 & e_7 has (v_3, v_4) are called as Parallel edge.

Simple & Multiple Graphs

Definition: A graph that has neither self loops or parallel edge is called as **Simple Graph** otherwise it is called as **Multiple Graph.**

For Example,

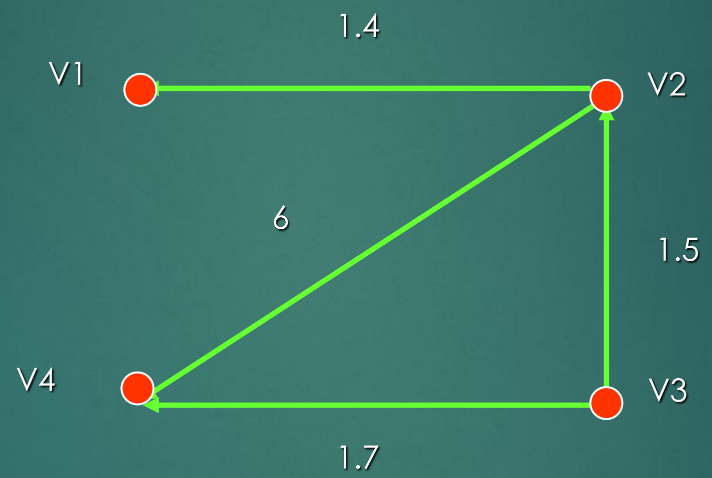
G1 (Simple Graph)

G2 (Multiple Graph)

Weighted Graph

Definition: If each edge or each vertex or both are associated with some +ve no. then the graph is called as **Weighted Graph**

For Example,



Adjacency & Incidence

Definition: Two vertices v_1 & v_2 vertices of G joins directly by at least one edge then there vertices called **Adjacent Vertices.**

For Example, In Graph G, v_1 & v_2 are adjacent vertices.

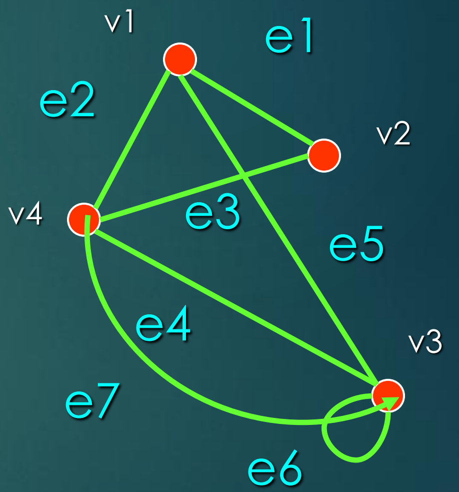
Definition: If V_i is end vertex of edge $e_{ij}=(v_i,v_j)$ then edge e_{ij} is said to be **Incident** on v_i. Similarly e_{ij} is said to be **Incident** on v_j.

For Example, In Graph G, e_1 is incident on v_1 & v_2.

Degree of a Vertex

Definition: The no. of edges incident on a vertex v_i with self loop counted twice is called as **degree of vertex vi.**

For Example, Consider the Graph G, $d(v_1)=3$,$d(v_2)=2$,$d(v_3)=5$,$d(v_4)= 4$



Definition :

A vertex with degree zero is called as **Isolated Vertex** & A vertex with degree one is called as **Pendant Vertex.**

Handshaking Lemma

Theorem: The graph G with e no. of edges & n no. of vertices, since each edge contributes two degree, the sum of the degrees of all vertices in G is twice no. of edges in G.

i.e. $\sum_{i=1}^{n} d(v_i)=2e$ is called as **Handshaking Lemma**.

Example: How many edges are there in a graph with 10 vertices, each of degree 6? **Solution:** The sum of the degrees of the vertices is 6*10 = 60. According to the Handshaking Theorem, it follows that 2e = 60, so there are 30 edges.

Matrix Representation of Graphs

A graph can also be represented by matrix.

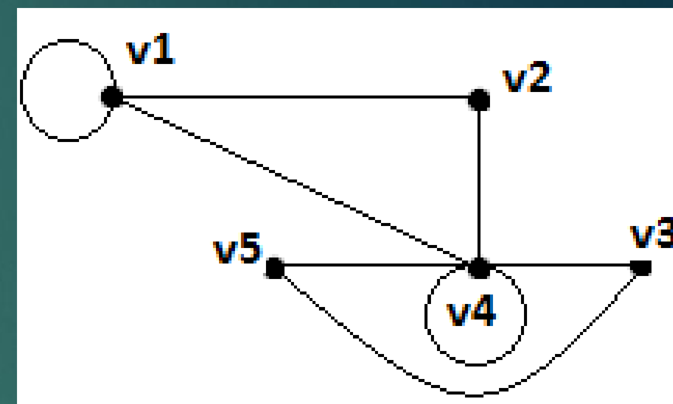
Two ways are used for matrix representation of graph are given as follows,

1. **Adjacent Matrix**
2. **Incident Matrix**

Lets see one by one…

1. Adjacent Matrix

The **A.M. of Graph G** with n vertices & no parallel edges is a symmetric binary matrix $A(G)=[a_{ij}]$ or order n*n where,



$a_{ij}=1$, if there is as edge between vi &vj.

$a_{ij}=0$, if v_i & v_j are not adjacent.

A self loop at vertex vi corresponds to $a_{ij}=1$.

For Example,

A(G)=

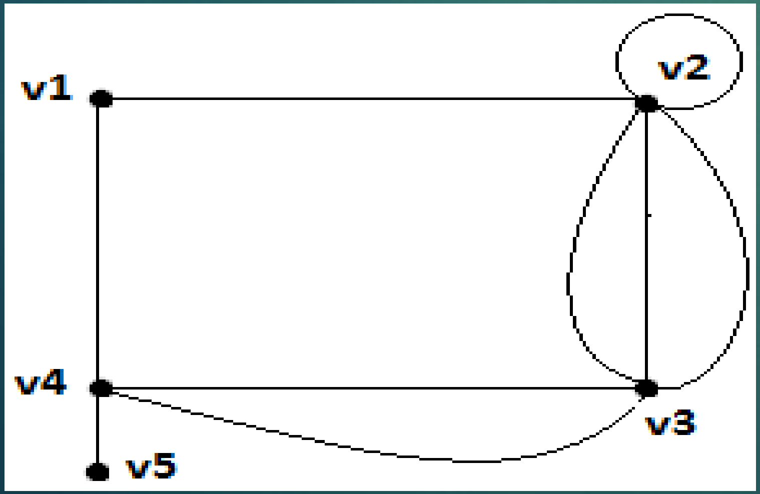
	v1	v2	v3	v4	v5
v1	1	1	0	1	0
v2	1	0	0	1	0
v3	0	0	0	1	1
v4	1	1	1	1	1
v5	0	0	1	1	0

1. Adjacent Matrix

The **A.M. of multigraph G** with n vertices is an n*n matrix $A(G)=[a_{ij}]$ where,

a_{ij}=N, if there one or more edge are there between v_i &v_j & N is no. of edges between v_i & v_j.

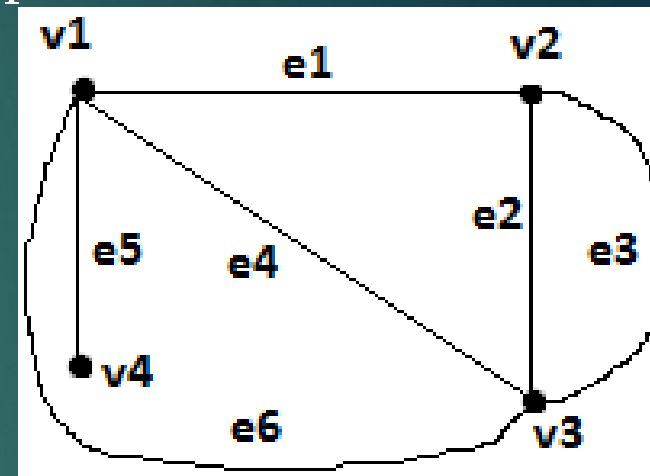
a_{ij}=0, otherwise.



	v1	v2	v3	v4	v5
v1	0	1	0	1	0
v2	1	1	3	0	0
v3	0	3	0	2	0
v4	1	0	2	0	1
v5	0	0	0	1	0

2. Incident Matrix

Given a graph G with n vertices , e edges & no self loops. The incidence matrix $x(G)=[X_{ij}]$ of the other graph G is an n*e matrix where,



$X_{ij}=1$, if j^{th} edge e_j is incident on i^{th} vertex v_i,

$X_{ij}=0$, otherwise.

Here n vertices are rows & e edges are columns.

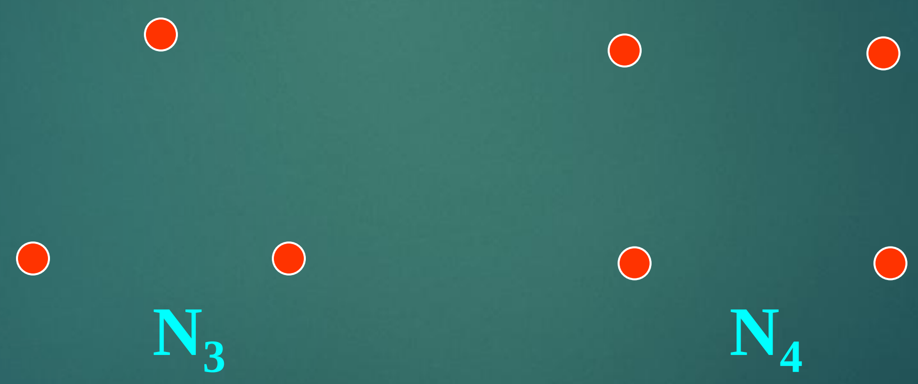
X(G)=

	e1	e2	e3	e4	e5	e6
v1	1	0	0	1	1	1
v2	1	1	1	0	0	0
v3	0	1	1	1	0	1
v4	0	0	0	0	1	0

Null Graph

Definition: If the edge set of any graph with n vertices is an empty set, then the graph is known as **null graph.**

It is denoted by N_n **For Example,**



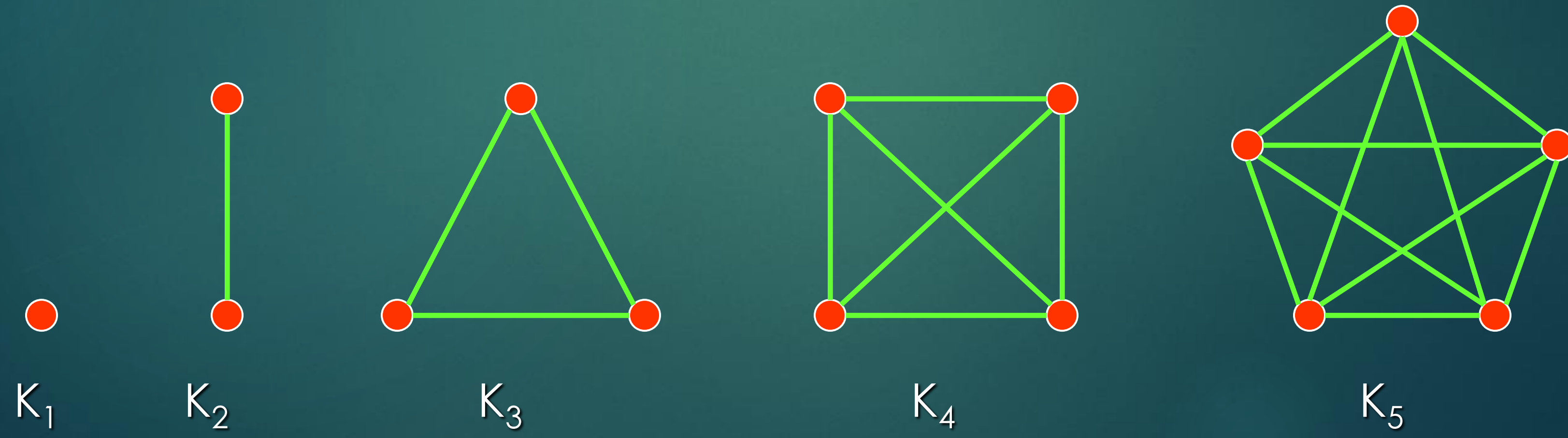
Complete Graph

Definition: Let G be simple graph on n vertices. If the degree of each vertex is (n-1) then the graph is called as **complete graph.**

Complete graph on n vertices, it is denoted by $\mathbf{K_n}$.

In complete graph K_n, the number of edges are

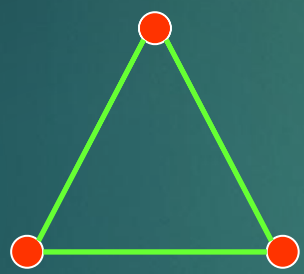
n(n-1)/2,For example,



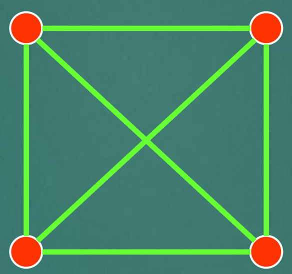
Regular Graph

Definition: If the degree of each vertex is same say ‘r’ in any graph G then the graph is said to be a **regular graph** of degree r.

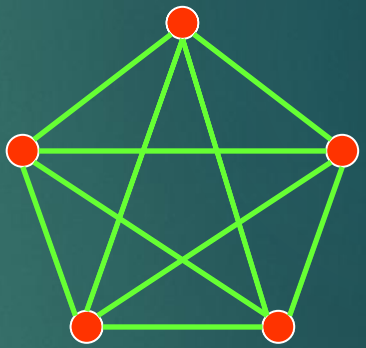
For example,



K_3



K_4



K_5

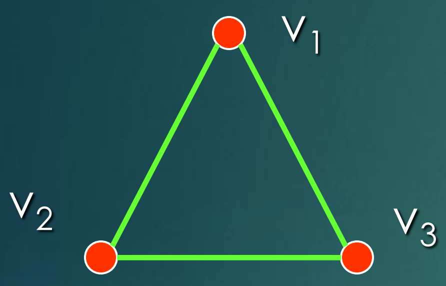
Bipartite Graph

Definition: The graph is called as **bipartite graph** , if its vertex set V can be partitioned into two distinct subset say V1 & V2. such that V1 U V2=V & V1 ∩ V2 = ∅ & also each edge of G joins a vertex of V1 to vertex of V2.

A graph can not have self loop.

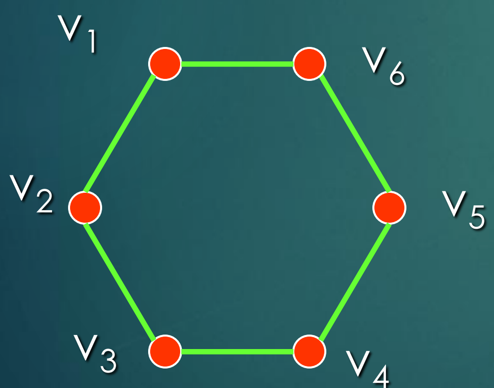
Bipartite Graphs

Example I: Is G1 bipartite?

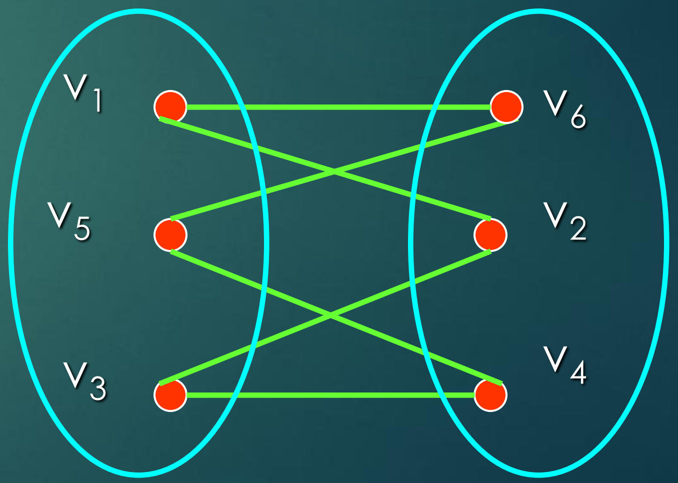


No, because there is no way to partition the vertices into two sets so that there are no edges with both endpoints in the same set.

Example II: Is G2 bipartite?



Yes, because we can display G2 like this:



Isomorphism

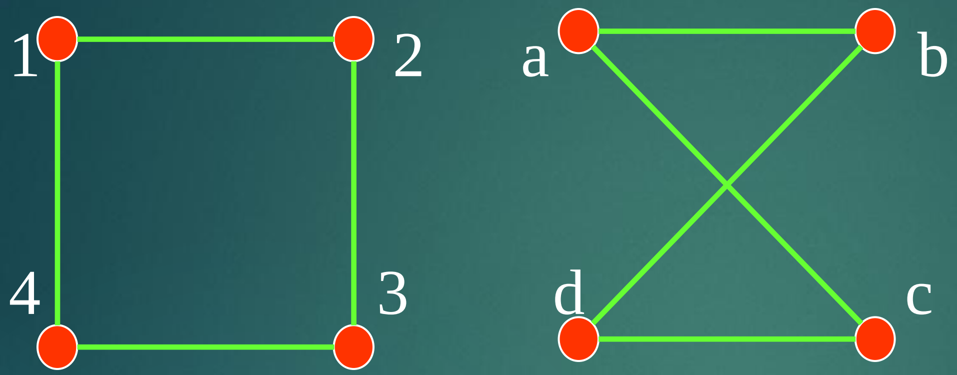
Definition: Two graphs are thought of as equivalent (**called isomorphic**) if they have identical behavior in terms of graph theoretic properties.

Two graphs G(V, E) & G'(V',E') are said to be **isomorphic** to each other if there is one-one correspondence between their vertices & between their edges such that incidence relationship in preserved.

It is denoted by G1=G2

Isomorphism

For Example



1	a
2	b
3	d
4	c

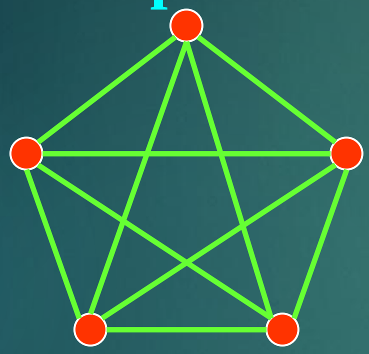
It is immediately apparent by definition of isomorphism that two isomorphic graphs must have,

- the same number of vertices,
- the same number of edges, and
- the same degrees of vertices.

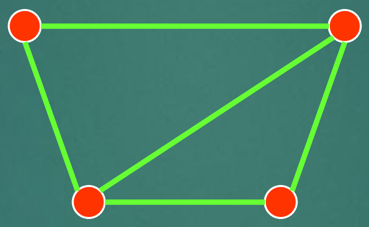
Sub Graph

Definition: A **sub graph** of a graph G = (V, E) is a graph G’ = (V’, E’) where V’⊆V and E’⊆E.

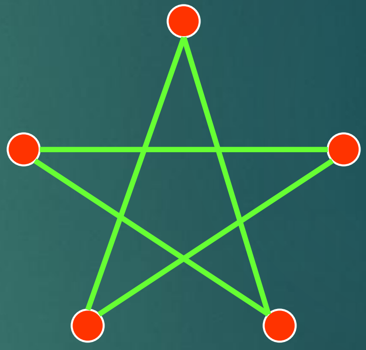
For Example:



G



G1

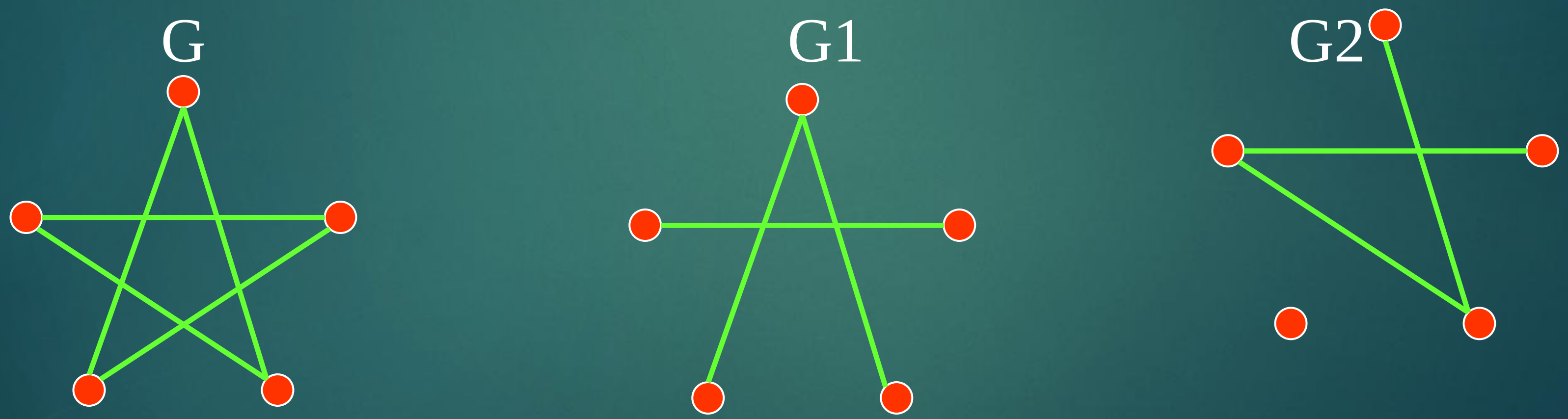


G2

Spanning Graph

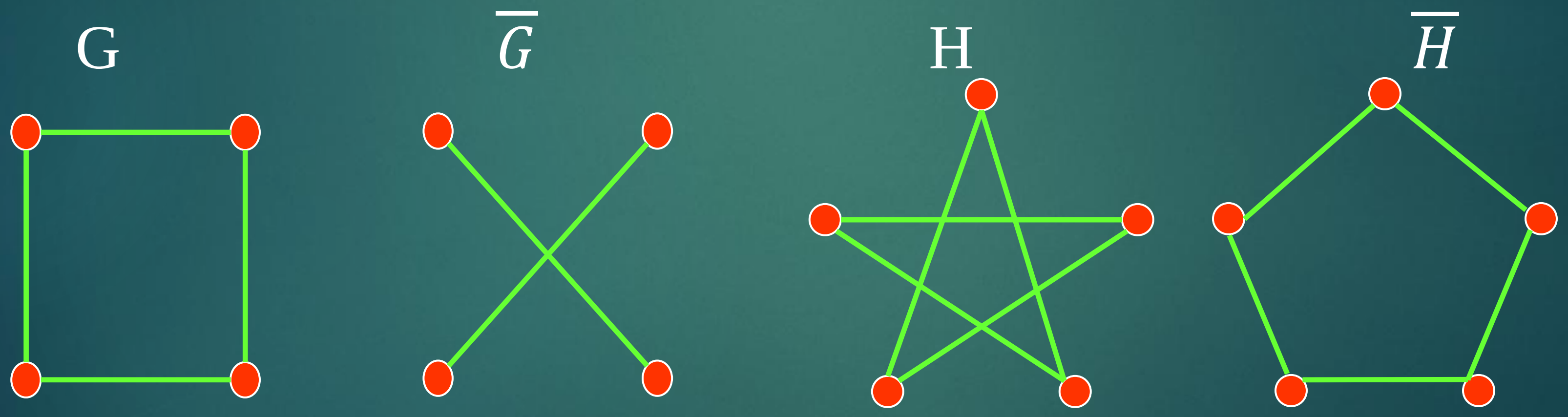
Definition: Let G=(V, E) be any graph. Then G’ is said to be the **spanning subgraph** of the graph G if its vertex set V’ is equal to vertex set V of G.

For Example:



Complement of a Graph

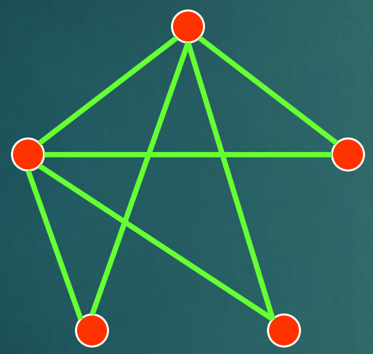
Definition: Let G is a simple graph. Then **complement of G** denoted by $\overline{G}$ is graph whose vertex set is same as vertex set of G & in which two vertices are adjacent if & only if they are not adjacent in G. **For Example:**



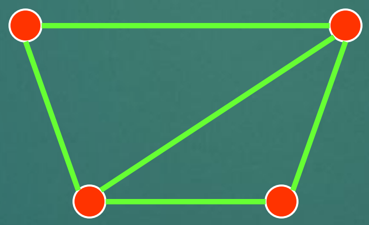
Operations on Graphs

Definition: The **union** of two simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the simple graph with vertex set $V_1 \cup V_2$ and edge set $E_1 \cup E_2$.

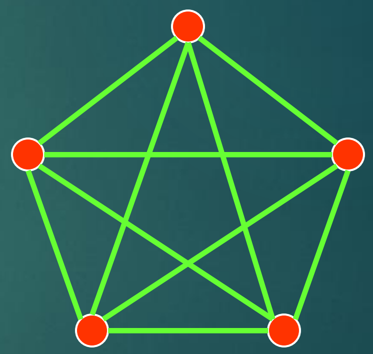
The union of G_1 and G_2 is denoted by $\mathbf{G_1 \cup G_2}$.



G_1



G_2

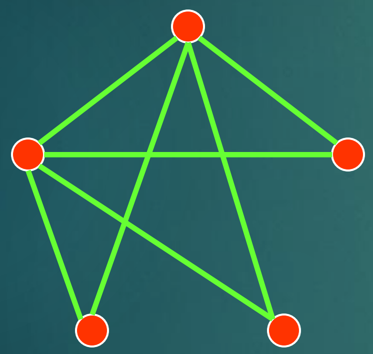


$G_1 \cup G_2$

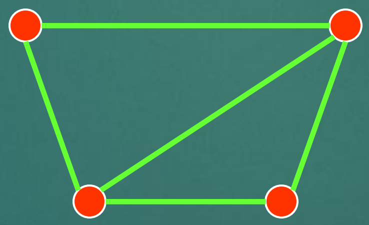
Operations on Graphs

Definition: The **Intersection** of two simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the simple graph with vertex set $V_1 \cap V_2$ and edge set $E_1 \cap E_2$.

The Intersection of G_1 and G_2 is denoted by $\mathbf{G_1 \cap G_2}$.



G_1



G_2

$G_1 \cap G_2$

THANKING YOU KEEP READING